STA 6467: Probability Theory II
Spring 2006
Problems for Assignment 8

Turn in exercise 34.3 in Billingsley and the first problem below (the second one is for practice, but you do not need to turn it in). To be clear, when doing Billingley’s exercise 34.3 you first must show that if $X$ and $Y$ are independent, then $E(Y|X) = E(Y)$. Next you must show that if $E(Y|X) = E(Y)$ (no independence assumed here), then $E(XY) = E(X)E(Y)$. For each part, state clearly what you must assume about the moments of $X$, $Y$, and/or $XY$.

1. Suppose that $E(|X|) < \infty$ and $E(|Y|) < \infty$ and that $E(Y|X) = X$ and $E(X|Y) = Y$. Show $X = Y$ almost surely.

2. Let $X$ and $Y$ be random variables on $(\Omega, \mathcal{F}, P)$, and suppose that $E[Y^2] < \infty$, and that

$$E[Y^2|\mathcal{G}] = X^2 \quad \text{and} \quad E[Y|\mathcal{G}] = X$$

for some $\sigma$-field $\mathcal{G} \subset \mathcal{F}$. Prove that $Y = X$ a.s.