1) Background:
- Shapes of valleys are described using observed cross-valley elevation profiles.
- There is interest to infer the phenomenon and processes that formed the valley.
- Glacial valleys are U-shaped, in contrast to V-shaped valleys, formed by other processes.
- A traditional null hypothesis (van der Meer, 1990).
- Approach to describing curvature.
- Previous models and regression models used in Geomorphology literature, most commonly used are versions of the power law regression model.
- Power law model for each valley separately (slope through value of exponent).
- Frequency variation of assumptions underlying model in q, heterogeneous.
- Method of data collection (of profiles) could pre-determine the conclusions reached.

2) Previous Work (Greenwood & Humphrey, 2002):
- Improved Sampling (new for Geomorphology literature).
- Within each valley, a relatively homogenous section is defined.
- ‘Blue’ ‘river’, line in columns 2 (Figure 1).
- A location on this center line is randomly selected.
- Always inferred to entire sampling region.

3) Data:
- Himalayan Valleys used to illustrate methods:
  - Kudi Kudi River Valley (red): Profile is U shaped.
  - Dhona Khola River Valley (blue): Profile is U shaped.
- Sampling from a Digital Elevation Map (DEM):
  - Profile inferred from DEM.
  - Observations may be too restrictive for ideal valley shape.
  - Models available may be too restrictive in shape.

4) Functional Data Analysis (FDA):
- Curve shape can be described through derivatives of the profile.
- Deriving to estimate the derivative is not recommended.
  - Noise in observations can generate derivative estimates difficult to interpret.
- Although a common practice, derivative estimation by difference is, in fact, seldom a good idea.
- Between U and V shape: approximated by two phase quadratic regression model.
- Also considering two phase linear regression model to allow asymmetric, but linear shape.

5) Cross-Validation type model selection criteria (Davies and Cavanaugh, 2002):
- Between U and V shape: approximated by two phase quadratic regression model.
- Also considering two phase linear regression model to allow asymmetric, but linear shape.

6) Drawbacks of Nonlinear regression:
- Smoothing for Kudi River profile was selected to provide visual smoothness (larger than CV selected).
- Incorporation of covariate information (direction of valley, bedrock type, etc.).

7) Modeling (discrete) 1st derivatives:
- Estimate of 1st derivatives at locations of observation.
- Model Selection amongst the previous regression models for 1st derivatives.
- Model Selection amongst the previous regression models for 1st derivatives.
- Model Selection amongst the previous regression models for 1st derivatives.
- Model Selection amongst the previous regression models for 1st derivatives.

8) Conclusions and Further Work:
- Nonlinear regression and model selection criteria provide a method for deriving curvature.
- FDA methods provide useful estimates of derivatives.
  - Information about the curvature displayed and wall height not as influential (in the visual description of derivatives).
- Between U and V shape, Dhona Khola between U and V shape (more linear than Kudi Kudi River valley).
- Model selection for the shape of 1st derivatives has potential for description/ comparison of curvature.
- Advantages of FDA methods over nonlinear regression:
  - Nonlinear regression and model selection criteria provide a method for deriving curvature.
  - FDA methods provide useful estimates of derivatives.
  - Information about the curvature displayed and wall height not as influential (in the visual description of derivatives)
  - Between U and V shape, Dhona Khola between U and V shape (more linear than Kudi Kudi River valley).
  - Model selection for the shape of 1st derivatives has potential for description/ comparison of curvature.
- Some Future Work:
  - Multiple profiles from a valley for use in Functional Linear Modeling (change in shape over the length of the valley).
  - Incorporation of covariate information (direction of valley, bedrock type, etc.).