# The History of Statistical Science: A Textbook View

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#### Outline and brief summary

- Motivation: This topic drew my interest from writing a textbook designed to teach the foundations of statistical science to students who plan to be data scientists.
- How did textbooks portray Statistics, before and after Fisher and Neyman? How did the focus change from 1900 to 1970, as Statistics matured as a field? What should be emphasized in the modern data science era?
- Yule's An Introduction to the Theory of Statistics (1911) was the standard until textbooks authored by Wilks, Kendall, and Cramér after World War 2 more fully portrayed modern mathematical statistics.
- Bayesian approach was promoted in books by Jeffreys and Savage, but only later in textbooks.
- Books on statistical methods by Fisher and Snedecor had greater impact on scientists using Statistics.

## George Udny Yule and his contributions

- George Udny Yule (1871-1951) began working in University College, London, in 1893 as demonstrator for Karl Pearson.
- Lectured at University College from 1902 until 1912, published An Introduction to the Theory of Statistics (1911). His research accomplishments should be better known:
  - Showed how to apply least squares to calculate partial regression coefficients and introduced partial correlation.
  - Proposed ways to measure association for discrete data, including advocating odds ratio; critical of Pearson's approach of assuming continuous bivariate distributions underlie two-way contingency tables.
  - Introduced modern approach to time series analysis, including devising correlogram and laying foundation for autoregressive modeling.

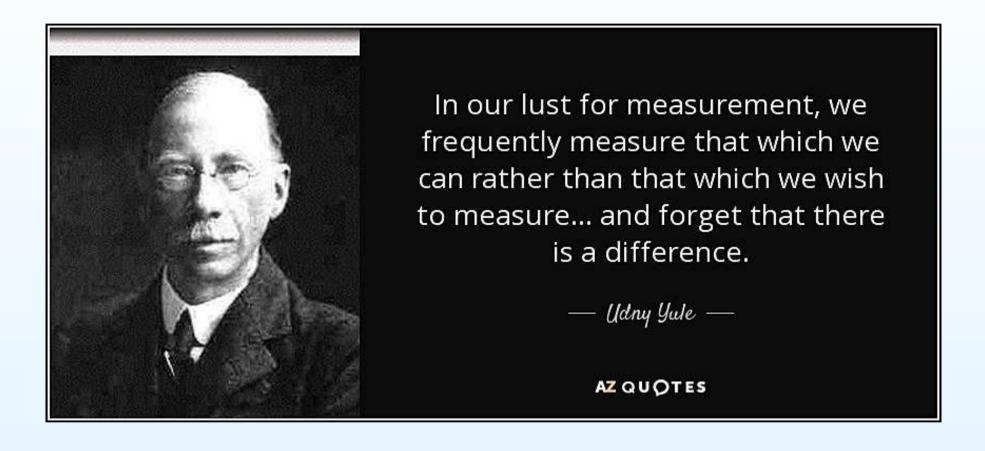
## Yule's An Introduction to the Theory of Statistics (1911)

Chapter title	Pages
1. Notation and terminology	7-16
2. Consistence	17-24
3-4. Association, partial association	25-59
5. Manifold classification	60-74
6. The frequency-distribution	75-105
7-8. Averages, Measures of dispersion, etc.	106-156
9-10. Correlation, Practical applications and methods	157-206
11. Miscellaneous theorems involving the use of the correlation-coefficient	207-224
12. Partial correlation	225-249
13. Simple sampling of attributes	250-271
14. Simple sampling continued: Removing the limitation of simple sampling	272-286
15. The binomial distribution and the normal curve	287-312
16. Normal correlation	313-330
17. The simpler cases of sampling for variables: Percentiles and mean	331-351

#### More on Yule's Introduction to the Theory of Statistics

- Karl Pearson was highly critical of Yule's book.
- Pearson and Heron, Biometrika (1913), "If Mr. Yule's views are accepted, irreparable damage will be done to the growth of modern statistical theory. ... Unthinking praise has been bestowed on a text-book which at many points can only lead statistical students hopelessly astray."
   (> 150 pages!)
- Yule left University College, moving in 1912 to the University of Cambridge for the remainder of his career.

#### G. Udny Yule



## Yule's book continued by Maurice Kendall

- A chance meeting in the Cambridge library with Maurice Kendall (1907-1983) resulted in Kendall's continuing to revise Yule's book, from 11th edition in 1937 until final 700-page-long 14th edition in 1950.
- However, all editions have only a superficial mention of Fisher, nothing on maximum likelihood or Neyman–Pearson theory of hypothesis testing.
- Jerzy Neyman on 1937 edition: "Yule's *Introduction* was the first book on statistics that came into my hands" (in 1916). "In my opinion, this is the best book on the theory of statistics that was ever written." Yet, Neyman also wrote, "We may regret only that it is an elementary introduction ... and not the theory itself. This still remains to be written."

#### Early Mathematical Statistics in North America

- In 1918 Carl J. West, mathematics professor at Ohio State University, published *Introduction to Mathematical Statistics*.
  - Perhaps the first occurrence of the term "mathematical statistics" in a book title, but focuses more on calculations than theory.
- Henry Rietz (1875-1943), in mathematics department at University of Iowa, was a founding member of the Institute of Mathematical Statistics in 1935 and its first president.
  - In 1927 Rietz published Mathematical Statistics, of higher mathematical level, often using differential and integral calculus.
  - But Rietz's book has no discussion of statistical inference, such as Fisher's landmark work in 1922 on likelihood functions and estimation.

#### Statistical Methods: R. A, Fisher (1925)

- Ronald Aylmer Fisher (1890-1962) started work in 1919 at Rothamsted Experimental Station, which led naturally to Statistical Methods for Research Workers (1925), with 14 editions to 1970.
- First presentation in book form of maximum likelihood, t tests, analysis of variance, and randomization and blocking in design of experiments.
- Book intended as laboratory notebook for scientists rather than student textbook, and Fisher included real-data examples as template for use of methods.

## Contents of R. A. Fisher's *Statistical Methods* (1925)

Chapter title	Pages
1. Introduction	1-26
2. Diagrams	27-42
3. Distributions	43-76
4. Tests of goodness of fit, independence and homogeneity, with table of $\chi^2$	77-100
5. Tests of significance of means, differences of means, regression coefficients	101-137
6. The correlation coefficient	138-175
7. Intraclass correlations and the analysis of variance	176-210
8. Further applications of the analysis of variance	211-233

Chapter 1, although called "Introduction," explains consistent, efficient, and sufficient statistics and method of maximum likelihood.

#### R. A. Fisher



To consult the statistician after an experiment is finished is often merely to ask him to conduct a post mortem examination. He can perhaps say what the experiment died of.

— Ronald Fisher —

AZ QUOTES

## George Snedecor's Statistical Methods (1937)

- Snedecor (1881-1974) taught courses in Statistics at Iowa State beginning in 1915 and in 1933 formed and directed the Statistical Laboratory.
- Topics covered in Snedecor's book were greatly influenced by Fisher, including Fisher's 1935 book on experimental design.
- Snedecor was one of first U.S. statisticians to appreciate the importance of Fisher, inviting him for extended visits to lowa State in 1931 and 1936 for summer courses and arranging for Fisher's first honorary doctorate in 1936.

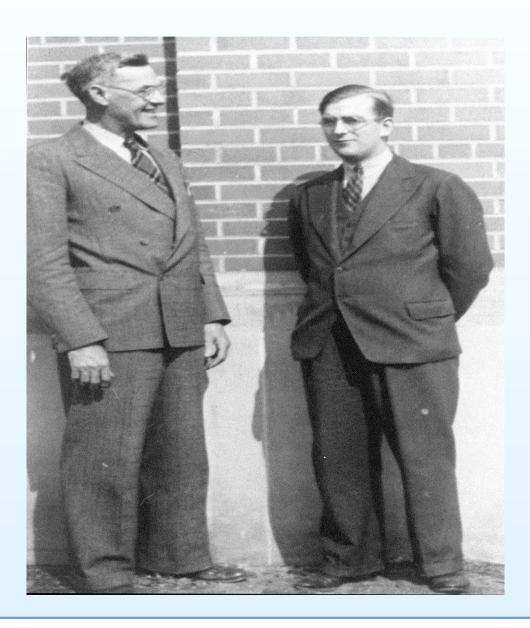
# Contents of Snedecor's Statistical Methods (1937)

Chapter title	Pages
1. Experiments on attributes	3-21
2. An experiment designed to compare measurements of individuals	22-42
3. Sampling from a normally distributed population	43-62
4. An experiment designed to compare two groups	63-74
5. Short cuts and approximations	75-88
6-7. Linear regression, correlation	89-133
8. Large sample theory	134-153
9. Enumeration data with multiple degrees of freedom	154-170
10-12. Analysis of variance and covariance	171-241
13. Multiple regression and covariance	242-263
14. Curvilinear regression	264-290
15. Individual degrees of freedom	291-316
16. Large samples of enumeration data. Binomial and Poisson distributions	317-334

#### Snedecor and Cochran: Statistical Methods

- Depth and coverage improved when Snedecor added William Cochran as co-author beginning with 5th edition in 1956.
- Cochran was Fisher's successor at Rothamsted, left in 1939 for Iowa State. His theoretical understanding and research with experimental designs and surveys made him an excellent complement.
- The final 8th edition of Snedecor and Cochran appeared in 1989.

# George Snedecor and William Cochran



#### World War 2 period: Modern mathematical statistics

- By beginning of World War 2, lack of established book on statistical theory and the Fisher and Neyman–Pearson advances paralleled lack of recognition of Statistics as field.
- Harold Hotelling (1940): "There is no comprehensive treatise on theoretical statistics, starting with first principles.
   ... The seeker after truth regarding statistical theory must make his way through or around an enormous amount of trash and downright error."
- This soon changed, due to landmark math stat textbooks by Samuel Wilks, Maurice Kendall, and Harald Cramér.

#### Samuel S. Wilks: Mathematical Statistics

- Wilks (1906-1964) received PhD at Univ. of Iowa in 1931, with Henry Rietz supervising, two years in England with Karl Pearson, appointment in mathematics at Princeton.
- Founding member of Institute of Mathematical Statistics in 1935 and editor of Annals of Mathematical Statistics from 1938 until 1949.
- Landmark publication on the large-sample chi-squared distribution of the likelihood-ratio statistic appeared in Annals of Mathematical Statistics in 1938.
- Wilks's 1937 notes evolved into lithoprinted *Mathematical Statistics* (1943), later published in 1947 by Princeton University Press.

## Contents of Wilks's *Mathematical Statistics* (1943)

Chapter title	Pages
1. Introduction	1-4
2. Distribution functions	5-46
3. Some special distributions	47-78
4. Sampling theory	79-97
5. Sampling from a normal population	98-121
6. On the theory of statistical estimation	122-146
7. Tests of statistical hypotheses	147-156
8. Normal regression theory	157-175
9. Normal regression theory and analysis of variance	176-199
10. On combinatorial statistical theory	200-225
11. An introduction to multivariate statistical analysis	226-270

# Samuel S. Wilks



#### Maurice Kendall: Advanced Theory of Statistics (1943)

- Originally planned in 1938 as joint project with M. S. Bartlett, J. Irwin, E. S. Pearson, and J. Wishart. After outbreak of World War 2, Kendall had to proceed alone.
- Kendall produce a two-volume work (Volume 1, 1943;
   Volume 2, 1946) of nearly 1000 pages during WW2!
- E-mail from Sir David Cox (2021): "Kendall was a man of phenomenal energy. Kendall volume 1 was largely written in air-raid shelters. At the same time MGK had a demanding full-time job in shipping control. World War 2 had the effect of a massive increase in interest in academic statistics, in Cambridge, Oxford, Imperial in the UK and Harvard, Berkeley, Columbia, Stanford, etc. In my own case it is highly unlikely that I would have become a statistician without the War, and I am one of many."

# Kendall's Advanced Theory of Statistics, Vol. 1 (1943)

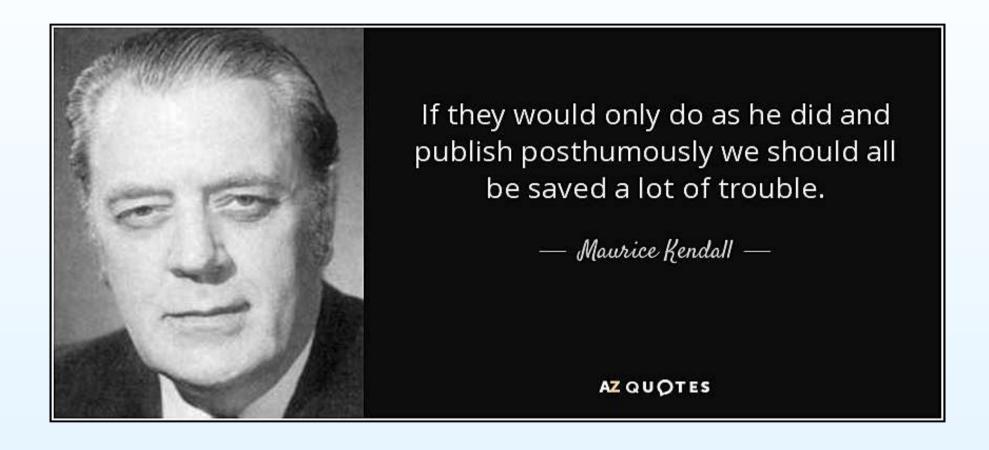
Chapter title	Pages
1. Frequency distributions	1-28
2. Measures of location and dispersion	29-48
3. Moments and cumulants	49-89
4. Characteristic functions	90-115
5-6. Standard distributions (1) (2)	116-163
7. Probability and likelihood	164-185
8. Random sampling	186-203
9. Standard errors	204-230
10. Exact sampling distributions	231-253
11. Approximations to sampling distributions	254-289
12. The $\chi^2$ distribution	290-307
13. Association and contingency	308-323
14. Product moment correlation	324-367
15. Partial and multiple correlation	368-387
16. Rank correlation	388-437

## Kendall's *Advanced Theory of Statistics*, Vol. 2 (1946)

Chapter title	Pages
17. Estimation: Likelihood	1-49
18. Estimation: Miscellaneous methods	50-61
19. Confidence intervals	62-84
20. Fiducial inference	85-95
21. Some common tests of significance	96-140
22. Regression	141-174
23-24. The analysis of variance (1) (2)	175-246
25. The design of sampling inquiries	247-268
26-27. General theory of significance tests (1) (2)	269-327
28. Multivariate analysis	328-362
29-30. Time series (1) (2)	363-439

## Bayesian approach?

## Maurice Kendall (about Bayesian statisticians)



## Cramér's Mathematical Methods of Statistics (1946)

- Other than Wilks's textbook and Kendall's volumes, the landmark publication in statistical theory immediately following the war was Harald Cramér's *Mathematical Methods of Statistics* (1946).
- Harald Cramér (1893-1985) was Professor of Actuarial Mathematics and Mathematical Statistics and director of Institute of Mathematical Statistics at University of Stockholm and President of university from 1950 to 1961. The book was based on his Stockholm lectures since 1930.
- Compared to existing books on statistical theory, mathematical level is higher, with much more emphasis on probability.

# Cramér's Mathematical Methods of Statistics (1946)

Chapter group	Pages
Part 1: Mathematical introduction	
1-3. Sets of points	3-18
4-7. Theory of measure and integration in $R_1$	19-75
8-9. Theory of measure and integration in $\mathcal{R}_n$	76-88
10-12. Various questions	89-136
Part 2: Random variables and probability distributions	
13-14. Foundations	137-165
15-20. Variables and distributions in $R_1$	166-259
21-24. Variables and distributions in $R_n$	260-322
Part 3: Statistical inference	
25-26. Generalities	323-340
27-29. Sampling distributions	341-415
30-31. Tests of significance, I	416-472
32-34. Theory of estimation	473-524
35-37. Tests of significance, II	525-556

# Harald Cramér



#### Mathematical Statistics in Quarter Century after WW2

- Statistics as a field received more attention in the form of university departments. See *Strength in Numbers: The Rising of Academic Statistics Departments in the U.S.* by A. Agresti and Xiao-Li Meng (2013) for impact, especially in 1960s, of growth due to baby boomers.
- As textbooks by Wilks, Kendall, and Cramér received attention, others appeared of slightly lower technical level.
- Two successful books that had much use as textbooks were Alexander Mood's Introduction to the Theory of Statistics (1950) and Introduction to Mathematical Statistics (1959) by Robert Hogg and Allen Craig.

#### Subject-specific texts in quarter century after WW2

William Cochran and Gertrude Cox (1950) Experimental Designs

William Cochran (1953) Sampling Techniques

Theodore Anderson (1958) *An Introduction to Multivariate Statistical Analysis* 

Erich Lehmann (1959) Testing Statistical Hypotheses

Henry Scheffé (1959) The Analysis of Variance

C. R. Rao (1965) Linear Statistical Inference and Its Applications

Norman Draper and Harry Smith (1967) Applied Regression Analysis

Thomas Ferguson (1967) Mathematical Statistics: A

Decision-Theoretic Approach

George E. P. Box and Gwilym Jenkins (1970) *Time Series Analysis:* Forecasting and Control

David R. Cox (1970) Analysis of Binary Data.

#### Gertrude Mary Cox

These authors all deserve their own page, such as Gertrude Cox—helped form Statistics Lab at Iowa State, first head of Statistics Dept. and Institute of Statistics at North Carolina State (Snedecor: "If you would consider a woman, I would recommend Gertrude Cox"), founding editor of *Biometrics*, first head of Statistics Research at Research Triangle Institute, President of ASA and Biometric Society.



#### Statistical Science from a Bayesian Perspective

- Cambridge professor Harold Jeffreys (1891-1989) *Theory of Probability* (1939) was a foundational Bayesian textbook.
- Title misleading, as book presents estimation and significance tests based on Laplace's theory of *inverse* probability, focusing on probabilities of parameters given data using probabilities of data given parameters.
- Fisher's maximum likelihood estimate justified as indistinguishable for large n from estimate using inverse probability with uniform prior distribution.
- Treatment of significance tests uses Bayes factors, which his article introduced four years before.
- R. A. Fisher: Jeffreys "makes a logical mistake on the first page which invalidates all the 395 formulae in his book."

## Savage The Foundations of Statistics

- U. of Chicago professor Leonard Jimmy Savage (1917-1971) published *The Foundations of Statistics* in 1954.
- Starting with six axioms and using rigorous proofs, Savage put forth a theory of subjective and personal probability and developed the theory of utility of von Neumann and Morgenstern and optimal decision making.
- Savage (1961) stated he was "too deeply in the grip of frequentist tradition to do a thorough job." Credited 1959 textbook by Robert Schlaifer of Harvard Business School as first written "entirely and wholeheartedly from the Bayesian point of view".

#### Schlaifer Probability & Statistics for Business Decisions

- Robert Schlaifer (1914-1994) self taught himself Statistics so he could teach it, decided the prevailing Pearson/Fisher/Neyman synthesis was inadequate for business applications.
- "The long-run frequency with which a certain method of making statements would produce incorrect statements is of no real interest to anyone."
- Two years later, Schlaifer co-wrote Applied Statistical Decision Theory, with Howard Raiffa at Harvard, which developed the idea of using conjugate prior and posterior distributions to simplify analyses computationally.

#### Future of Statistical Theory in Data Science World

- We have focused on contributions in first 70 years of 20th century that helped statisticians and scientists more fully understand this rapidly-developing field.
- With revolution instigated by Tukey (1977) in data analysis expansion of Statistics and by Breiman (2001) with algorithmic alternative to modeling culture, statistical theory increasingly viewed as a small part of greater data science.
- Viewing Statistics as the science of learning from data, perhaps it's increasingly artificial to place boundary between "mathematical statistics" and methods of data analysis and to have purely a frequentist or Bayesian focus.

#### Present/Future of Statistical Theory Textbooks

- For a textbook now on the theoretical foundations of Statistics / Data Science, what should be included?
- In the recent book Foundations of Statistics for Data Scientists, with R and Python (2022), my co-author Maria Kateri and I thought it important to supplement the standard topics with many examples employing R and Python and with simulations using them and apps (such as at www.artofstat.com) to illustrate concepts, while avoiding boundaries between theory/methods and frequentist/Bayes.
- My talk has been based on article invited by *Brazilian Journal of Probability and Statistics* (2021). In it, I asked several esteemed statisticians (e.g., David Cox, Brad Efron, Susan Murphy, Xiao-Li Meng) to contribute their thoughts about relevance of foundations of Statistics in modern data science era.

#### Conclusion

Article on which this talk is based available at

www.stat.ufl.edu/~aa/articles/Agresti\_BJPS.pdf

Grazie mille a Regina e Annie per l'invito e grazie a tutti voi per la partecipazione!

Thanks for taking the time to attend this talk!